

Localization of graviphoton and graviscalar on the brane

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Abstract

The question of whether the Kaluza-Klein (KK) graviphoton $h_{5\mu}$ and graviscalar h_{55} are localized or not on the brane is one of the important issues. In this letter, we address this problem in five dimensions. Here we consider the massless (zero-mode) propagations without requiring the Z_2 -symmetry on $h_{5\mu}$. We obtain the graviton $h_{\mu\nu}$, graviphoton, and graviscalar exchange amplitudes on shell. We find that the graviscalar has a tachyonic mass. It turns out that $h_{5\mu}$ admits the localized zero-modes on the brane while h_{55} does not have a localized zero-mode. This is contrasted to the fact that the bulk spin-0 field has a localized zero-mode on the brane but the bulk spin-1 field does not have a localized solution in five dimensions.

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In the conventional Kaluza-Klein (KK) approach of the five dimensional (5D) gravity, the spacetime manifold is factorized as $M_4 \otimes S^1$. Here M_4 is the Minkowski spacetime and S^1 is the circle. The spectrum of the 5D pure gravity is split into four dimensional (4D) massless fields such as graviton, graviphoton, graviscalar, and an infinite tower of massive spin-two fields [1,2]. In particular, the $U(1)$ gauge symmetry of the graviphoton in the 4D effective action is originated from the translational isometries in the extra dimension.

On the other hand, there have been much interest in the phenomenon of localization of gravity proposed by Randall and Sundrum (RS) [3,4]. There have been developed a large number of brane world models afterwards [5–7]. RS assumed a single positive tension 3-brane and a negative bulk cosmological constant in the 5D spacetime [4]. They have obtained a 4D localized gravity by fine-tuning the tension of the brane to the cosmological constant. The introduction of branes usually gives rise to the warping of the extra dimensions, resulting in the non-factorizable spacetime. Apparently, the presence of the brane breaks the translational isometries in the extra dimensions. Hence we worry about obtaining the graviphoton in the RS approach.

It is not easy to derive the zero-mode propagations in the non-factorizable compactification. In the conventional KK approach, one can find the zero mode $f(x)$ only by requiring $\partial_5 f = 0$ in the equation of motion. This is possible because it is in the factorizable compactification. In this case the 5D Laplacian is split into the 4D Laplacian \square and ∂_5^2 , the latter produces a (mass)²-term [2]. Equivalently the 4D effective action for zero-modes can be obtained after the integration of the 5D pure gravity action over $x^5 = z \in S^1$. However, in the non-factorizable compactification, there exist additional terms that are function of $x^5 = z \in R$ in the linearized equation. In general we cannot obtain the consistent zero-mode solution only by requiring $\partial_5 f = 0$ in the level of the equation of motion. The integration of the 5D action over z is a good starting point to obtain the zero-mode solution for the non-factorizable compactification [8–11]. Also the study of this issue is very important for the phenomenological purposes because its zero modes (massless modes) correspond to the standard model particles localized on the brane. If the brane world scenario is correct, the various fields we observe are the zero-modes of the KK [12] and bulks fields [13] which are trapped on the brane by the gravitational interaction ¹. Here we consider the KK fields only.

A simple choice for the KK fields is a part ($h_{5\mu} = h_{55} = 0$) of the RS gauge² to obtain the localized 4D gravity on the brane [4]. The brane-bending effect appears under the RS gauge with the localized matter source [14]. Here the bending of the wall $\hat{\xi}^5$ acts like a new scalar under the RS gauge³. Ivanov and Volovich [15] and Myung and Kang [16] have discussed the propagation of $h_{5\mu}$ with $h_{55} = 0$. The propagation of all metric components

¹The field theoretic mechanism for gauge field localization on a brane was first suggested in [19]. Also the localization of quantum fields on the brane was recently discussed [20].

²This is composed of the Gaussian-Normal gauge : ($h_{5\mu} = h_{55} = 0$) and the 4D transverse traceless gauge : $\partial_\mu h^{\mu\nu} = 0, h^\mu_\mu(h) = 0$.

³For the other approach, see ref. [21] .

including $h_{5\mu} \neq 0, h_{55} \neq 0$ was investigated in [17]. It turned out that the massive modes of $h_{5\mu}, h_{55}$ with uniform external sources cannot propagate on the branes. Recently, the case of $h_{55} \neq 0, h_{5\mu} = 0$ with the localized source was discussed [18]. However, it was pointed out that at long distance where we can obtain the 4D gravity, the propagation of h_{55} is not allowed. The next question is whether the massless modes of graviphoton $h_{5\mu}$ and graviscalar h_{55} propagate or not on the brane.

It is known for $h_{55} \sim \varphi$ on the RS brane that the consistency with $h_\rho^\rho = h_{5\mu} = 0$ requires $h_{55} = 0$ without the external source [11]. Concerning the massless propagation of $h_{5\mu} \sim a_\mu$, we expect that the breaking of isometries in the extra dimension by the brane makes the 4D effective action not being invariant under U(1) gauge transformations manifestly [3]. Explicitly, it comes from the Z_2 -symmetry argument. This is based on the fact that RS ground state solution is symmetric under $z \rightarrow -z$. If one requires that this symmetry be preserved up to the linearized level, $h_{\mu\nu}(x, z), h_{55}(x, z)$ are even with respect to z , but $h_{5\mu}$ is odd : $h_{5\mu}(x, -z) = -h_{5\mu}(x, z)$. This implies that $h_{5\mu}(x, 0) = 0$ on the brane. Thus we do not expect to have the zero-modes of the gravivector. Here we do not require such a Z_2 -symmetry on the linearized calculation. Then the analysis of the linearized equation around the RS background reveals that the graviphoton possesses the U(1) gauge symmetry [12].

In this paper, we clarify whether the graviphoton $h_{5\mu}$ and graviscalar h_{55} are localized or not on the brane with the matter sources. This is one of the important issues about the brane world scenario [9, 8, 22–24]. The naive condition that the zero-mode is localized on the brane is equivalent to the normalizability of the ground state wave function on the brane [9]. This requires that after the integration of the 5D action over z it be finite [9, 13]. However, this is valid for the bulk fields. Actually a further work is necessary for a complete study of the massless propagations including the graviphoton and graviscalar on the brane. As a definite criterion, we introduce the local sources to calculate the graviton, graviphoton, and graviscalar exchange amplitudes on shell.

We start from the second RS model with a single brane at $z = 0$ [4, 12]

$$I = \int d^4x \int_{-\infty}^{\infty} dz \frac{\sqrt{-\hat{g}}}{16\pi G_5} (\hat{R} - 2\Lambda) - \int d^4x \sqrt{-\hat{g}_B} \sigma. \quad (1)$$

Here G_5 is the 5D Newtonian constant, Λ is the bulk cosmological constant, \hat{g}_B is the determinant of the induced metric for the 3-brane, and σ is the tension of the brane. We assume that the value of σ is fine-tuned such that $\Lambda = -6k^2 (< 0)$ with $k = 4\pi G_5 \sigma / 3$. Let us introduce the domain-wall metric,

$$\begin{aligned} ds^2 &= \hat{g}_{MN} dx^M dx^N = H^{-2}(z) g_{MN} dx^M dx^N \\ &= H^{-2}(z) [\gamma_{\mu\nu} dx^\mu dx^\nu + \Phi^2 (dz - \kappa A_\mu dx^\mu)^2]. \end{aligned} \quad (2)$$

Here $H = k|z| + 1$, $\Phi^2 = g_{55}$, and $-\kappa\Phi^2 A_\mu = g_{5\mu}$. The above corresponds to the standard KK decomposition as

$$g_{MN} = \begin{pmatrix} \gamma_{\mu\nu} + \kappa^2 \Phi^2 A_\mu A_\nu & -\kappa \Phi^2 A_\mu \\ -\kappa \Phi^2 A_\nu & \Phi^2 \end{pmatrix} \quad (3)$$

with $A^\mu = \gamma^{\mu\nu} A_\nu$ and $A \cdot A = A_\mu A^\mu$. Here κ is introduced for the small gauge coupling constant.

Under the specific class of coordinate transformations such as

$$x^\mu \rightarrow \tilde{x}^\mu = \tilde{x}^\mu(x), \quad z \rightarrow \tilde{z} = z + \xi(x), \quad (4)$$

we obtain from $\tilde{g}_{MN} = \frac{\partial x^P}{\partial \tilde{x}^M} \frac{\partial x^Q}{\partial \tilde{x}^N} g_{PQ}$ as

$$\tilde{\gamma}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial x^\beta}{\partial \tilde{x}^\nu} \gamma_{\alpha\beta}, \quad \tilde{A}_\mu = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} A_\alpha + \kappa^{-1} \frac{\partial \xi}{\partial \tilde{x}^\mu}, \quad \tilde{\Phi}(\tilde{x}, \tilde{z}) = \Phi(x, z) \quad (5)$$

We observe that $\gamma_{\mu\nu}$ transforms like a 4D metric tensor and Φ a scalar field under diffeomorphisms in Eq. (4). Also we point out that the 5D diffeomorphisms are split into the 4D diffeomorphisms plus the U(1) gauge transformations for A_μ .

In this work, we are mainly interested in the zero mode effective action. It is a non-trivial problem to determine what the zero mode is if the full spacetime is not factorizable. In order to obtain the zero modes, we assume that $\gamma_{\mu\nu}$, A_μ , and Φ are functions of x -coordinates only. Plugging Eqs.(2) and (3) with $\gamma_{\mu\nu}(x)$, $A_\mu(x)$, and $\Phi(x)$ into Eq.(1) and integrating it over z leads to [12]

$$I_{zero-m} = \frac{1}{16\pi G_4} \int d^4x \sqrt{-\gamma} \left[\Phi R(\gamma) - \frac{\kappa^2}{4} \Phi^3 F^2 + 6k^2 \left(\Phi^{-1} + \Phi - 2\sqrt{|\delta_\nu^\mu + \kappa^2 \Phi^2 A^\mu A_\nu|} + \kappa^2 \Phi A \cdot A \right) \right]. \quad (6)$$

We observe that the zero-mode gravitational degrees of freedom in the 5D spacetime are split into the 4D graviton $\gamma_{\mu\nu}$, a graviphoton A_μ , and a graviscalar Φ as usual. However, the properties of the vector field and the scalar field are very different from those in the conventional KK reduction. The first two terms in Eq. (6) are the same form as in the conventional KK compactification, and thus they have the U(1) gauge symmetry. The difference from the conventional KK approach is the last term which is proportional to k^2 . If we start from the KK metric decomposition with $A_\mu = 0$ and $\Phi = 1$ in Eq. (3) as in the RS approach, this “potential” term disappears and one obtains the pure 4D gravity without the cosmological constant on the brane. The zero cosmological constant arises because of the fine-tuning between the brane tension σ and the 5D bulk cosmological constant Λ .

Apparently the non-linear term ($\sqrt{|\delta_\nu^\mu + \kappa^2 \Phi^2 A^\mu A_\nu|}$) as well as the squared term $A \cdot A$ imply not only that the 4D effective action no longer has the U(1) gauge symmetry, but also that the graviphoton does not exist. This phenomena arises mainly from the presence of the brane in the 5D Anti de Sitter spacetime. However, this observation is not a complete one. Actually the propagation of fields should be determined by the perturbation analysis around the RS solution. As will see later, the non-linear term and $A \cdot A$ cannot generate any mass-like term.

In order to see explicitly how the dynamical aspect of Φ comes out, let us conformally transform the metric as

$$\gamma_{\mu\nu} \rightarrow \bar{\gamma}_{\mu\nu} = \Phi \gamma_{\mu\nu}. \quad (7)$$

The zero-mode effective action Eq. (6) is then written by

$$I_{zero-m}^E = \frac{1}{16\pi G_4} \int d^4x \sqrt{-\bar{\gamma}} \left[R(\bar{\gamma}) - \frac{\kappa^2}{4} \Phi^3 F^2 - \frac{3}{2} \Phi^{-2} \bar{\nabla}^\mu \Phi \bar{\nabla}_\mu \Phi + 6k^2 \Phi^{-2} \left(\Phi^{-1} + \Phi - 2\sqrt{|\delta_\nu^\mu + \kappa^2 \Phi^3 A^\mu A_\nu|} + \kappa^2 \Phi^2 A \cdot A \right) \right] \quad (8)$$

in the Einstein frame. Here all contractions are done using the metric $\bar{\gamma}^{\mu\nu}$ as $F^2 = \bar{\gamma}^{\mu\nu}\bar{\gamma}^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}$, and $A \cdot A = \bar{\gamma}^{\mu\nu}A_\mu A_\nu$. Hence $\bar{\gamma}^{\mu\nu}$ corresponds to the canonical metric. Now we wish to derive the equations from the effective action (8). First of all, we have to change the non-linear term of $\sqrt{|\delta_\nu^\mu + N_\nu^\mu|}$ with $N_\nu^\mu = \kappa^2\Phi^3 A^\mu A_\nu$ into a manageable form. Considering the small gauge coupling constant ($\kappa < 1$), we assume that $\delta_\nu^\mu > N_\nu^\mu$. Using the formula as

$$\sqrt{\det[\mathbf{1} + \mathbf{x}]} = 1 + \frac{1}{2}\text{tr}(\mathbf{x}) - \frac{1}{4}\text{tr}(\mathbf{x}^2) + \frac{1}{8}(\text{tr}(\mathbf{x}))^2 + \dots, \quad (9)$$

one finds

$$\sqrt{|\delta_\nu^\mu + N_\nu^\mu|} = 1 + \frac{1}{2}\kappa^2\Phi^3 A \cdot A - \frac{1}{8}(\kappa^2\Phi^3 A \cdot A)^2 + \dots \quad (10)$$

For simplicity we use here the relation of

$$\sqrt{|\delta_\nu^\mu + N_\nu^\mu|} \simeq 1 + \frac{1}{2}\kappa^2\Phi^3 A \cdot A. \quad (11)$$

Even if we use this instead of Eq. (10), we never lose the generality for analyzing the RS background. To check it, we have the relations

$$\frac{\delta\sqrt{|\delta_\nu^\mu + N_\nu^\mu|}}{\delta\bar{\gamma}^{\mu\nu}} = \frac{1}{2}\kappa^2\Phi^3 A_\mu A_\nu (1 - \frac{1}{2}\kappa^2\Phi^3 A \cdot A + \dots), \quad (12)$$

$$\frac{\delta\sqrt{|\delta_\nu^\mu + N_\nu^\mu|}}{\delta\Phi} = \frac{3}{2}\kappa^2\Phi^2 A \cdot A (1 - \frac{1}{2}\kappa^2\Phi^3 A \cdot A + \dots), \quad (13)$$

$$\frac{\delta\sqrt{|\delta_\nu^\mu + N_\nu^\mu|}}{\delta A^\mu} = \kappa^2\Phi^3 A_\mu (1 - \frac{1}{2}\kappa^2\Phi^3 A \cdot A + \dots). \quad (14)$$

Making use of Eq. (11), we obtain the truncated equations of motion

$$R_{\mu\nu} = \frac{\kappa^2}{2}\Phi^3 \left(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}\bar{\gamma}_{\mu\nu}F^2 \right) - 6k^2\kappa^2(1 - \Phi)A_\mu A_\nu + \frac{3}{2}\Phi^{-2} \left[\bar{\nabla}_\mu \Phi \bar{\nabla}_\nu \Phi - 2k^2\bar{\gamma}_{\mu\nu}(\Phi^{-1} + \Phi - 2) \right], \quad (15)$$

$$\bar{\nabla}^\mu F_{\mu\nu} + 12k^2\kappa^2\Phi^{-3}(1 - \Phi)A_\nu = -3\Phi^{-1}\bar{\nabla}^\mu \Phi F_{\mu\nu}, \quad (16)$$

$$\bar{\nabla}^\mu \bar{\nabla}_\mu \Phi - \Phi^{-1}\bar{\nabla}^\mu \Phi \bar{\nabla}_\mu \Phi + 2k^2 \left[\Phi^{-1}(4 - \Phi - 3\Phi^{-1}) - \kappa^2\Phi^2 A \cdot A \right] = \frac{\kappa^2}{4}F^2. \quad (17)$$

It is easily checked that $A_\mu = 0$ and $\Phi = 1$ satisfies Eqs. (16) and (17). Also if we use the full expression Eq. (10) for the non-linear term, counting Eqs. (13) and (14) with $A_\mu = 0$ leads to the same situation. In this case, Eq. (15) leads to $R_{\mu\nu} = 0$. Considering Eq. (12) and (10), we find $R_{\mu\nu} = 0$. Thus any 4D Ricci-flat metric $\bar{\gamma}_{\mu\nu}$ is a solution to the 4D effective action Eq. (8). In particular, $\bar{\gamma}_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-+++)$ corresponds to the RS solution with Z_2 -orbifold symmetry. It is noted that only for the case of $A_\mu = 0$, we can find the consistent solution. This is because the case of $A_\mu \neq 0$ results in the unwanted situation. That is, it is not easy to find a solution to the equations including a lot of terms like $A \cdot A$, $(A \cdot A)^2$, \dots . This is why in the RS approach they set $A_\mu = 0$ at the beginning [4].

Now we are in a position to consider the perturbation analysis around the RS ground state solution ($\bar{\gamma}_{\mu\nu} = \eta_{\mu\nu}$, $A_\mu = 0$, $\Phi = 1$). Let us introduce the small fluctuations around the RS solution

$$\gamma_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad A_\mu = 0 + a_\mu, \quad \Phi = 1 + \kappa\varphi. \quad (18)$$

Considering $g_{MN} = \eta_{MN} + \kappa h_{MN}$, we have the relations: $h_{5\mu} = -a_\mu$, $h_{55} = 2\varphi$. We note that the non-zero a_μ breaks the Z_2 -symmetry. From Eq.(7), one finds

$$\bar{\gamma}_{\mu\nu} = \eta_{\mu\nu} + \kappa \bar{h}_{\mu\nu}, \quad \bar{h}_{\mu\nu} = h_{\mu\nu} + \varphi \eta_{\mu\nu}. \quad (19)$$

Then the bilinear action of Eq. (8) which governs the perturbative dynamics is given by [2,25]

$$\begin{aligned} I_{zero-m}^{bil} = & \frac{\kappa^2}{16\pi G_4} \int d^4x \left\{ -\frac{1}{4} \left[\partial^\mu \bar{h}^{\alpha\beta} \partial_\mu \bar{h}_{\alpha\beta} - \partial^\mu \bar{h} \partial_\mu \bar{h} + 2\partial^\mu \bar{h}_{\mu\nu} \partial^\nu \bar{h} - 2\partial^\mu \bar{h}_{\mu\alpha} \partial^\alpha \bar{h}_\nu^\alpha \right] \right. \\ & - \frac{1}{2} (\partial_\mu a_\nu \partial^\mu a^\nu - \partial_\nu a_\mu \partial^\mu a^\nu) - \frac{3}{2} \partial_\mu \varphi \partial^\mu \varphi + 6k^2 \varphi^2 \\ & \left. + \frac{1}{2} h_{\mu\nu} T^{\mu\nu} + a_\mu J^\mu + \varphi J_\varphi \right\}, \end{aligned} \quad (20)$$

where $\bar{h} = \eta^{\mu\nu} \bar{h}_{\mu\nu} = h + 4\varphi$. Here we introduce the 4D external sources of $(T^{\mu\nu}(x), J^\mu(x), J_\varphi(x))$ to obtain the correct physical propagations. Originally these all belong to the localized sources on the brane as $(T^{\mu\nu}(x), J^\mu(x), J_\varphi(x))\delta(z)$ [14, 26]. But after the integration over z these lead to the last line of Eq. (20). Surprisingly, it turns out that although the Z_2 -symmetry along z -axis is broken at the linearized level, the bilinear effective action is invariant under the $U(1)$ gauge transformation. Our previous observation about the $U(1)$ symmetry breaking caused by the non-linear term and $A \cdot A$ is not correct at least for the RS ground state solution. Here a nice combination of the non-linear term and $A \cdot A$ in Eq. (8) does not generate any mass term like $a \cdot a$. This appears as higher order term than the squared order: $6k^2\kappa^2(1-\phi)A \cdot A \rightarrow -6k^2\kappa^3\varphi a \cdot a$. We expect that this may contribute to the quantum correction. However, if $\varphi = 0$, this term does not appear. The known method to obtain the localization of the zero-modes is find the bilinear action without the external sources [9,24,13]. The action Eq. (20) obtained after the integration over z and the perturbation around the RS background is finite. Also it seems to have the canonical forms for all fluctuation fields. Hence, following the conventional criterion, the zero-modes of the graviton, graviscalar, graviphoton all are localized on the brane. However, this may be wrong because it misses the roles of the potential terms and the external source. The actual propagation of the physical zero-modes on the brane can be justified by the calculation of their exchange amplitudes for the sources [27,2,25,28].

In order to understand what physical states there are, let us derive the linearized equations. From the action Eq. (20) we obtain the equations of motion

$$\square \bar{h}_{\mu\nu} + \partial_\mu \partial_\nu \bar{h} - (\partial_\mu \partial^\alpha \bar{h}_{\alpha\nu} + \partial_\nu \partial^\alpha \bar{h}_{\alpha\mu}) - \eta_{\mu\nu} (\square \bar{h} - \partial^\alpha \partial^\beta \bar{h}_{\alpha\beta}) = -T_{\mu\nu}, \quad (21)$$

$$\square a_\mu - \partial_\mu (\partial_\nu a^\nu) = -J_\mu, \quad (22)$$

$$\square \varphi + 4k^2 \varphi = -\frac{1}{3} J_\varphi. \quad (23)$$

Here we find the 4D diffeomorphisms plus the U(1) gauge symmetry. Hence these can be taken into account by the source conservation laws

$$\partial^\mu T_{\mu\nu} = 0, \quad \partial^\mu J_\mu = 0. \quad (24)$$

This means that the two equations Eqs. (21) and (22) are compatible with the source conservation laws. By taking the trace of Eq. (21), we have

$$\square \bar{h} - \partial^\alpha \partial^\beta \bar{h}_{\alpha\beta} = \frac{1}{2}T \quad (25)$$

with $T = T_\rho^\rho$. Hence Eq. (21) becomes

$$\square \bar{h}_{\mu\nu} + \partial_\mu \partial_\nu \bar{h} - \left(\partial_\mu \partial^\alpha \bar{h}_{\alpha\nu} + \partial_\nu \partial^\alpha \bar{h}_{\alpha\mu} \right) = -(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T). \quad (26)$$

So far we have not chosen any gauge for $\bar{h}_{\mu\nu}, a_\mu$. Now let us choose the harmonic gauge and Lorenz gauge, respectively

$$\partial^\mu \bar{h}_{\mu\nu} = \frac{1}{2}\partial_\nu \bar{h}, \quad \partial_\mu a^\mu = 0. \quad (27)$$

Using these gauge conditions, Eq. (26) and Eq. (22) reduce to

$$\square \bar{h}_{\mu\nu} = -(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T), \quad \square a_\mu = -J_\mu. \quad (28)$$

The first equation is exactly the same equation which was derived for the graviton zero-mode using the brane-bending effect [14]. In the brane-bending calculation, it requires $h = 0$. Furthermore we get from Eq. (25) and (28) the trace equation

$$\square \bar{h} = T. \quad (29)$$

To obtain the on-shell exchange amplitude induced by the sources, let us plug Eqs. (21), (22), and (23) into Eq. (20) [2]. Then we find

$$I_{zero-m}^{ampp} = \frac{\kappa^2}{32\pi G_4} \int d^4x \left\{ \frac{1}{2} \bar{h}_{\mu\nu}(x) T^{\mu\nu}(x) + a_\mu(x) J^\mu(x) + \varphi(x) J_\varphi(x) \right\}. \quad (30)$$

We wish to take its Fourier-transformed form which makes the calculation easy [27],

$$I_{zero-m}^{ampp} = \frac{\kappa^2}{32\pi G_4} \int d^4p \left\{ \frac{1}{2} \bar{h}_{\mu\nu}(p) T^{\mu\nu}(p) + a_\mu(p) J^\mu(p) + \varphi(p) J_\varphi(p) \right\}. \quad (31)$$

From Eqs. (28) and (23), their Fourier-transformed fluctuations are given by

$$\bar{h}_{\mu\nu}(p) = \frac{1}{p^2} [T_{\mu\nu}(p) - \frac{1}{2}\eta_{\mu\nu}T(p)], \quad (32)$$

$$a_\mu(p) = \frac{J_\mu(p)}{p^2}, \quad (33)$$

$$\varphi(p) = \frac{J_\varphi(p)}{3(p^2 + m_\varphi^2)} \quad (34)$$

with $m_\varphi^2 = -4k^2$. Substituting these into Eq. (31) leads to

$$I_{zero-m}^{ampp} = \frac{\kappa^2}{32\pi G_4} \int d^4p \left\{ \frac{1}{2p^2} (T^{\mu\nu}(p)T_{\mu\nu}(p) - \frac{1}{2}T^2(p)) + \frac{J^\mu(p)J_\mu(p)}{p^2} + \frac{J_\varphi^2(p)}{3(p^2 + m_\varphi^2)} \right\}. \quad (35)$$

In order to study the massless states, it is best to choose the light-cone frame of $p_\mu = (p_1, 0, 0, p_4)$ [2]. Then the source conservation law of $p^\mu T_{\mu\nu} = 0$ in Eq. (24) give us the relations

$$T_{11} = T_{41} = T_{44}, \quad T_{12} = T_{42}, \quad T_{13} = T_{43}. \quad (36)$$

Using this, one finds the spin-2 exchange amplitude

$$T^{\mu\nu}(p)T_{\mu\nu}(p) - \frac{1}{2}T^2(p) = |T_{+2}|^2 + |T_{-2}|^2, \quad (37)$$

where $T_{\pm 2} = \frac{1}{2}(T_{22} - T_{33}) \pm iT_{23}$. On the other hand, the source conservation law of $p^\mu J_\mu = 0$ leads to

$$J_1 = J_4. \quad (38)$$

Making use of this, one has the spin-1 exchange amplitude

$$2J^\mu J_\mu = |J_{+1}|^2 + |J_{-1}|^2 \quad (39)$$

with $J_{\pm 1} = J_2 \pm iJ_3$. Finally, plugging this information into Eq. (35) leads to

$$I_{zero-m}^{ampp} = \frac{\kappa^2}{62\pi G_4} \int d^4p \left\{ \frac{1}{p^2} (|T_{+2}|^2 + |T_{-2}|^2 + |J_{+1}|^2 + |J_{-1}|^2) + \frac{2|J_\varphi|^2}{3(p^2 + m_\varphi^2)} \right\}, \quad (40)$$

indicating that a total of four massless states: the gravitons with helicities $\lambda = \pm 2$ and the graviphotons with helicities $\lambda = \pm 1$ [2]. Therefore, it proves that $\bar{h}_{\mu\nu}$ and a_μ indeed represent the massless spin-2 propagations and the massless spin-1 propagations on the brane, respectively. Of course these all are localized on the brane.

On the other hand, the graviscalar φ in Eq. (40) appears to be massive. Unfortunately, it has the tachyonic mass $m_\varphi^2 = -4k^2$. It may indicate the unstable, massless mode on the brane. This means that the graviscalar cannot be localized on the brane. This can be easily read off from the 4D effective action in Eq. (6). Essentially we wish to treat the graviscalar as a massless freedom. However, the brane with tension $\sigma \sim k$ generates the unwanted tachyonic terms for the graviscalar Φ which are proportional to k^2 ⁴. In the absence of the sources, the consistency between the linearized equation with $h = 0$ leads to $\varphi = 0$ [11]. At this stage we comment on the role of the trace of $h_{\mu\nu}$. We assume that in the presence of the sources this is the pure gauge degree of freedom, as was in the brane-bending approach [14]. Therefore we can choose $h = 0$, which implies that Eq. (29) leads to $\square\varphi = T/4$. This

⁴ This may imply that our ansatz of $\gamma_{\mu\nu}(x), A_\mu(x), \Phi(x)$ is not appropriate for integrating out the massive KK modes. We thank Kakushadze for pointing out this.

leads to the contradiction to the graviscalar equation of Eq. (23). We have two equations (that is, one is massless, whereas the other is massive) for the same field φ . Hence we can choose $\varphi = 0$ on the brane for the consistency.

In conclusion we have established the localization of the graviphotons on the brane. Although these belong to asymmetric modes which do not satisfy the Z_2 -background symmetry, they possess $U(1)$ symmetry. Also the graviscalar is not a massless scalar field localized on the brane even for introducing the matter sources.

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